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Application of Mathematical Modeling in Population forecasting

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Abstract. In the present paper we have to study future population forecasting in India by the geometrical model used to census population data. Design is based on the projected population of India, estimated for the design period. Changes in population over the years occur, and the system should be designed into account of the population at the end of the design period. Factors affecting changes in population are increase due to births, decrease due to deaths, increase and decrease due to the migration and increase due to the annexation. The present and past population record in India can be obtained from the census population records. After collecting these population records, the population at the end of design period is predicted using various geometrical methods as suitable for the growth pattern.

Key words: Arithmetic mean, Geometric mean, logistic equation, growth rate etc.

1. Geometrical increase model representation

In this model, the average increase in future population forecasting per decade is calculated from the past census reports. This increase is added to the present population to find out of the population next decade. Thus it is assumed that the population is increasing at constant rate. Hence

(1.1) $\frac{dP}{dt} = C$ i.e., rate of change of population with respect to time is constant.

Therefore, Population after nth decade will be

(1.2) $P_n = P + nC$

Where, P_n is the population after 'n' decades and 'P' present population.

The percentage increase in population from decade to decade is assumed to remain constant. Geometric mean increase is used to find out the future increment in population. Since this process gives higher values and hence should be applied for a new industrial town at the beginning of development for only few decades. The population at the end of 'n^{th}' decade P_n can be estimated as

(1.3) $$P_n = P\left(1+\frac{I_G}{100}\right)^n$$

Where, I_G = Geometric mean (%)

$P =$ Present population, and

$n =$ Number of decades.

Now the modification of arithmetic increase process is suitable for an average size town under normal condition where the growth rate is found to be in increasing order. While adopting this process the increase in increment is considered for calculating future population. The incremental increase is determined for each decade from the past population and average value is added to the present population along with the average rate of increase.

Hence, the population after 'n^{th}' decade is

(1.4) $$P_n = P + nX + \left\{\frac{n(n+1)}{2}\right\}Y$$

Where, P_n = Population after 'n^{th}' decade

X = Average increase, and

Y =Incremental increase.

2. Graphical model representation

In this process, the populations of last few decades are correctly plotted to suitable scale on graph. The population curve is smoothly extended for getting future population. This extension should be done carefully and it requires proper experience and judgment. The best way of applying this process is to extend the curve by comparing with population curve of some other population having the growth condition.

3. Master Plan Model

The big and metropolitan cities are generally not developed in haphazard manner, but are planned and regulated by local bodies according to master plan. The master plan is prepared for next 25 to 30 years for the every city in India. According to the master plan the city is divided into various zones such as residence, commerce and industry. The population densities are fixed for various zones in the master plan. From this population density total water demand and wastewater generation for that zone can be worked out. By this method it is very easy to access precisely the design population.

4. Logistic curve Model

This process is used when the growth rate of population due to births, deaths and migrations takes place under normal situation and it is not subjected to any extraordinary changes like epidemic, war, earth quake or any natural disaster, etc., and the population follows the growth curve characteristics of living things within limited space and economic opportunity. If the population of India is plotted with respect to time, the curve so obtained under normal condition looks like S-shaped curve and is known as logistic curve.

A Mathematical solution of the first order Logistic equation is as

(5.1) $$\log_e\left(\frac{P_s - P}{P}\right) - \log_e\left(\frac{P_s - P_0}{P_0}\right) = -KP_s t$$

Where, P = Population at any time t from the origin

P_s = Population at the start point

K = Constant

t = Time in years.

From the above equation we get

(5.2) $$\log_e\left(\frac{P_s - P}{P}\right)\left(\frac{P_0}{P_s - P_0}\right) = -KP_s t$$

After solving we get,

(5.3) $$P = \frac{P_s}{1 + \frac{P_s - P_0}{P_0}\log_e^{-1}(-KP_s t)}$$

(5.4) Substituting $\frac{P_s - P_0}{P_0} = m$ (a constant)

(5.5) $-KP_s = n$ (another constant)

(5.6) $$P = \frac{P_s}{1 + m\log_e^{-1}(nt)}$$

This is the required equation of the Logistic curve, which will be used for predicting population. If only three pairs of the characteristic values P_0, P_1, P_2 at times $t = t_0 = 0$ t_1 and $t_2 = 2t_1$ extending over the past record are chosen, the saturation population P_s and constant m and n can be estimated by the equation, as follows

(5.7) $$P_s = \frac{2P_0P_1P_2 - P_1^2(P_0 + P_2)}{P_0P_2 - P_1^2}$$

(5.8) $$\frac{P_s - P_0}{P_0} = m$$

(5.9) $$n = \frac{2.3}{t_1}\log_{10}\left(\frac{P_0(P_s - P_1)}{P_1(P_s - P_0)}\right)$$

5. Iller Bankasi Model

It is a geometric increase method but calculation of Kg is different. In this method $Kg = C$

(6.1) $C = \left(\sqrt[a]{\frac{P_2}{P_1}} - 1 \right) x100$ where $a = t_2 - t_1$

If $C < 1$ then $C = 1$

If $1 < C < 3$ then $C = C_i$

If $C > 3$ then $C = 3$

To calculate C average

(6.2) $\bar{C} = \frac{\sum_{i=1}^{x} C_i}{x} = \frac{C_1 + C_2 + C_3 + ... + C_x}{x}$ where x = number of past record interval

The future population is calculated as

(6.3) $P_{future} = P_{past} \left(1 + \frac{\bar{C}}{100} \right)^n$ where $n = t_{future} - t_{past}$

6. Decreasing rate of increase

India has some limiting saturation population, and that its rate of growth is a function of its population deficit.

(7.1) $\frac{dP}{dt} = K_d (P_s - P)$

Where K_d is constant, S is the saturation population and P is the population.

K_d is calculated as

(7.2) $K_d = \frac{-\log\left(\frac{P_s - P_2}{P_s - P_1} \right)}{t_2 - t_1}$

K_d Average is calculated as

(7.3) $\bar{K}_d = \frac{\sum_{i=1}^{x} (K_d)_i}{x}$

At last, future population is calculated as

(7.4) $P_{future} = P_{past} + (P_s - P_{last})(1 - e^{-K_d (t_{future} - t_{last})})$

7. Solution of the Geometrical increase model

Predict the population growth of India per decade is the future population forecast in the 10^{th} year difference. Population per decade is calculated from the past census reports by using the equation (1.1). This increase is added to the present population to find out of the population next decade. In accordance by the process of the geometrical mean model, the future population forecast will be increase 1.7 billion in the year 2051.

Population growth of India per decade			
Census Year	**Population**	**Increment**	**Change %**
1951	361,088,000	-	-
1961	439,235,000	7,81,47,000	21.6
1971	548,160,000	10,89,25,000	24.8
1981	683,329,000	13,51,69,000	24.7
1991	846,387,888	16,30,58,888	23.9
2001	1,028,737,436	18,23,49,548	21.5
2011	1,210,726,932	18,19,89,496	17.7
	Average increment	**12,52,67,739**	

Future population forecast for year 2021 is, $P_{2021} = 1,210,726,932 + 12,52,67,739 \times 1$
$= 1,33,59,94,671$

Similarly, $P_{2031} = 1,210,726,932 + 12,52,67,739 \times 2$
$= 1,461,262,410$

$P_{2041} = 1,210,726,932 + 12,52,67,739 \times 3$
$= 1,586,530,149$

$P_{2051} = 1,210,726,932 + 12,52,67,739 \times 4$
$= 1,711,797,888.$

And again Geometric mean increase is applied to obtain the future increment in the Indian future population growth per decade. Now the above population data in predict the population forecasting for the year 2021, 2031, 2041 and 2051 using the Geometrical progression model.

Population growth of India per decade			
Census Year	**Population**	**Increment**	**Geometrical increase rate of growth**
1951	361,088,000	-	-
1961	439,235,000	7,81,47,000	7,81,47,000/361,088,000 = 0.21
1971	548,160,000	10,89,25,000	10,89,25,000/439,235,000 = 0.24
1981	683,329,000	13,51,69,000	13,51,69,000/548,160,000 = 0.24
1991	846,387,888	16,30,58,888	16,30,58,888/683,329,000 = 0.23
2001	1,028,737,436	18,23,49,548	18,23,49,548/846,387,888 = 0.21
2011	1,210,726,932	18,19,89,496	18,19,89,496/1,028,737,436 = 0.17

Population for Geometric mean $I_G = (0.21 \times 0.24 \times 0.24 \times 0.23 \times 0.21 \times 0.17)^{1/6}$

$= (0.000099320256)^{1/6}$

$= 0.215198697$ i.e., 21.51%

Future Population forecasting by G.M. in the year 2021 is

$$P_{2021} = 1,210,726,932 \times (1 + 0.215198697)^{1}$$

=1,471,273,790.

Similarly for the year 2031, 2041 and 2051 can calculated by the G.M. are

$$P_{2031} = 1,210,726,932 \times (1 + 0.215198697)^{2}$$

= 1,787,889,992,

$$P_{2041} = 1,210,726,932 \times (1 + 0.215198697)^{3}$$

=2,172,641,589

And $$P_{2051} = 1,210,726,932 \times (1 + 0.215198697)^{4}$$

= 2,640,191,229.

Now the census population data are predicted the future population forecasting for the year 2021, 2031, 2041 and 2051 by using the modification of the arithmetical increase model. This model is simply for an average size under the normal condition, where the growth rate is obtained to be an increase model for calculating the future population.

Population growth of India per decade			
Census Year	**Population**	**Increment (X)**	**Incremental increase (Y)**
1951	361,088,000	-	-
1961	439,235,000	7,81,47,000	-
1971	548,160,000	10,89,25,000	3,07,78,000
1981	683,329,000	13,51,69,000	2,62,44,000
1991	846,387,888	16,30,58,888	2,78,89,888
2001	1,028,737,436	18,23,49,548	1,92,90,660
2011	1,210,726,932	18,19,89,496	-3,60,052
	Total	84,96,38,932	10,38,42,496
	Average	12, 52, 67,739	2,07,68,499

Future population forecasting in the year 2021 is

$$P_{2021} = 1,210,726,932 + (12,52,67,739 \times 1) + \left\{ \frac{1(1+1)}{2} \right\} \times 2,07,68,499$$

= 1,356,763,170

$$P_{2031} = 1,210,726,932 + (12,52,67,739 \times 2) + \left\{ \frac{2(2+1)}{2} \right\} \times 2,07,68,499$$

= 1,523,567,907

$$P_{2041} = 1,210,726,932 + (12,52,67,739 \times 3) + \left\{ \frac{3(3+1)}{2} \right\} \times 2,07,68,499$$

= 1,711,141,143

$$P_{2051} = 1,210,726,932 + (12,52,67,739 \times 4) + \left\{ \frac{4(4+1)}{2} \right\} \times 2,07,68,499$$

=1,919,482,878.

8. Calculate for Logistic Model in the estimated Population data

The estimated population in three years can be the equation (5.7) and

Year	Population	Saturation population P_s
1961	454 176 666	773680449
1971	560 274 314	
1981	705 395 576	
1991	879 557 823	1941390993
2001	1 062 684 631	
2011	1 239 215 258	

P_0 (1961) = 454 176 666 $t_0 = 0$

P_1 (1971) = 560 274 314 $t_1 = 10$ years

P_2 (1981) = 705 395 576 $t_2 = 20$ years.

The saturation population can be calculated by using the equation (5.7) we have

$$P_s = \frac{2 \times 454\,176\,666 \times 560\,274\,314 \times 705\,395\,576 - (560\,274\,314)^2 (454\,176\,666 + 705\,395\,576)}{454\,176\,666 \times 705\,395\,576 - (560\,274\,314)^2}$$

= 773680449

And we can calculate the population forecasting and using the equation (5.7), we have

P_0 (1991) = 879 557 823 $t_0 = 0$

P_1 (2001) = 1 062 684 631 $t_1 = 10$ years

P_2 (2011) = 1 239 215 258 $t_2 = 20$ years.

$$P_s = \frac{2 \times 879\,557\,823 \times 1\,062\,684\,631 \times 1\,239\,215\,258 - (1\,062\,684\,631)^2 (879\,557\,823 + 1\,239\,215\,258)}{879\,557\,823 \times 1\,239\,215\,258 - (1\,062\,684\,631)^2}$$

= 1 941 390 993

By using the equation (5.8) and (5.9) we have,

$$m = \frac{1941390993 - 879\,557\,823}{879\,557\,823} = 1.207$$

$$n = \frac{2.3}{10} \log_{10} \left(\frac{879\,557\,823(1941390993 - 1\,062\,684\,631)}{1\,062\,684\,631(1941390993 - 879\,557\,823)} \right)$$

= - 0.0378

By using the equation (5.6) we can calculate the Population in 2021, we have

$$P = \frac{1941390993}{1 + 1.207 \log_e^{-1}(-0.0378 \times 30)} = 1398341427$$

9. Comparative Graphical model

In this process the census populations of India already developed under similar conditions are plotted. The curve of past population of the India under consideration is plotted on the same graph. The curve is extended carefully by comparing with the population curve having the similar condition of growth. The advantage of this process is that the future population can be predicted from the present population even in the absence of some of the past census report.

Discussion: Forecasting of population can be accomplished with short term different mathematical models by using present and past population records that can be obtained the future population. Graphical projection of the past population growth curve continuing whatever trends the historical data indicate the future population can be predicted by plotting the population of other methods. The process to be compared should be as similar as possible to census data being studied. Main reasons of the population change are birth, death and migration. Where information regarding births and deaths is available, the natural increase can be easily estimated the population of India. When calculating, population forecasting should be calculated by the different geometrical model, logistic model and Iller Bankasi model; otherwise it will not affect the number of future population census recorded.

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